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In conclusion, Mr. Smith submitted that it would be desirable to have careful drawings, and, where practicable, rubbings also, made of all such existing monuments, in order that these most interesting memorials, which contain valuable confirmations of written documents, as well as curious illustrations of the manners and customs of bygone times, may be preserved from oblivion; and stated that he would be much gratified by receiving any communications on the subject, though they went no further than to state the existence of such crosses, in order to complete the materials for a general history of these Christian memorials, so deeply interesting, even in an historical point of view alone.

Rev. Charles Graves, F.T.C.D., read a Memoir, by Mr. George Boole, of Lincoln, on Discontinuous Functions.

The author deduces in succession three theorems for the expression of the discontinuous function, $f(x)$. The first theorem, which is free from signs of integration, implies that between the limits $x = a$, and $x = a + \Delta a$,

$$f(x) = \frac{1}{\pi}\left(\tan^{-1}\frac{a + \Delta a - x}{k} - \tan^{-1}\frac{a - x}{k}\right) f(x), \qquad (1)$$

provided that we suppose k a positive quantity, and take the limit to which the second member approaches, as k approximates to 0. When $x = a$, or $a + \Delta a$, the first member of the above equation must be divided by 2; and when x transcends those limits, the first member is to be replaced by 0. From this formula, the author deduces his second theorem, involving one sign of integration, viz.:

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{k\,da\,f(a)}{k^2 + (a - x)^2}, \qquad (2)$$

in the second member of which the limits $-\infty$ and ∞ may be replaced by any other real limits, p and q, when all the values of x, for which $f(x)$ does not vanish, lie between the limits p and q. This theorem is subject to the same conditions, with reference to the interpretation of the second member, and to the changes of value which it undergoes in passing the limits, as the preceding one.

By an integral transformation, Mr. Boole then deduces the third, or Fourier's theorem, involving two signs of integration, viz.:

$$f(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} da\, dv\, \varepsilon^{-kv}\cos(av-xv)f(a). \qquad (3)$$

He remarks, that when this theorem is written in the form,

$$f(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} da\, dv \cos(av-xv)f(a),$$

we must attach to the symbol $\int_0^\infty$ a meaning different from its ordinary one, and understand by $\int_0^\infty dv\, \phi(v)$, the limit of $\int_0^\infty dv\, \varepsilon^{-kv}\phi(v)$, for decreasing positive value of k. Mr. Boole proposes to designate an integral of this kind as taken in a *limiting* sense, and he observes that some anomalous results have been obtained by writers who have neglected the distinction here implied. Thus, from the equations

$$\int_0^\infty dx \cos x = 0, \qquad \int_0^\infty dx \sin x = 1,$$

in which the sign $\int_0^\infty$ has been used in its limiting sense, have been deduced, by taking that symbol in its ordinary sense, and integrating without reference to the factor understood, ε^{-kx}, the incompatible conclusions,

$$\cos\infty = 0, \ \sin\infty = 0.$$

Mr. Boole remarks that, by a converse error, other writers have been led to infer the incorrectness of Fourier's theorem.

From Fourier's theorem the author finally deduces a theorem for the discontinuous expression of $\frac{f(x)}{t^n}$, viz.:

$$\frac{f(x)}{t^n} = \frac{1}{\pi\Gamma(n)}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} da\,dv\,dw \cos\left((a-x)v - tw + \frac{n\pi}{2}\right)w^{n-1}f(a), \quad (4)$$

in which the symbols $\int_0^\infty$ are both used in a limiting sense. This theorem, it is observed, is susceptible of important applications, in the theory of definite multiple integrals.

As respects the expression of discontinuity, the formulæ (1), (2), (3), are shewn to be equivalent. The advantage which the second possesses over the first is, that x enters into the second member in a rational form; and the advantage of the third, or Fourier's theorem, over them both, is, that x enters into the second member exponentially, which affords facilities for both the direct and the inverse processes of the differential calculus.

In the fourth theorem the author remarks that both x and t enter exponentially.

Sir William R. Hamilton made some observations on Mr. Boole's communication.

Professor Harrison made some remarks on the peculiarities of the anatomy of the Emu. [This paper, not having been received in proper time for insertion here, will be printed in the Appendix, No. VII.]

Professor Allman announced the addition of Deotis Maritima to the Irish Fauna. This plant he had observed during the latter part of the summer of 1845, in the sand-hills in the neighbourhood of Dungarvan, county of Wexford.

DONATIONS.

Memorie della Reale Academia della Scienza di Torenzo. Serie Secunda. Tome. VI. Presented by the Academy.